

# Optimized Evaluation of Students Performances Using Fuzzy Logic

E. Sakthivel, K. Senthamarai Kannan and S. Arumugam

**Abstract**— Fuzzy Inference System for student academic performance evaluation is based on Fuzzy Logic Techniques (FLT). In this system student performance depends on exam paper results and it is evaluated only as success or failure. The study proposes a new performance evaluation method based on the Fuzzy Logic System (FLS). The study proposes a new evaluation method to find a performance of students results based an FLS and also compared with the Classical Method (CL). Students' performance based on organizing technique at Manonmaniam Sundaranar University in the Department of Statistics was carried out using FLT and it was compared with the CL. Twenty students took part for the statistical course were considered as study samples. Several approaches using FLT have been proposed a practical method for evaluating student academic performance and compared with existing statistical method. In this paper, evaluation of the results showed variations between the CL and FLMS.

**Keywords:** Fuzzy Logic, Trapezoidal fuzzy number, Student Academe Performance and Classical Methods.

## 1 INTRODUCTION

THE fuzzy logic tool was introduced in 1965 by Lotfi Zadh, which is a mathematical tool for dealing . It offers a soft computing partnership which is the important concept of computing with words. It provides a technique to deal with imprecision and information granularity. The fuzzy theory provides a mechanism for representing linguistic constructs such as many, low, medium, often few. In general, the fuzzy logic provides an inference structure that enables appropriate human reasoning capabilities. On the contrary, the traditional binary set theory describes crisp events, that either do or do not occur. It uses probability theory to explain an which occurs by event measuring the chance with which a given event is expected to occur. The fuzzy logic theory is based upon the notion of relative graded membership which are the functions of mentation and cognitive processes. The utility of fuzzy sets lie in their ability to model uncertainty or ambiguous data. Konstantina chrysafiadi an Viruou (2012) have evaluated the effectiveness and accuracy of the student model of a web-based educational environment for teaching computer programming. Yadav. S.R and Singh P. V (2012) has studied the rule based Fuzzy Expert System which automatically converts the crisp data into fuzzy set and also calculates the total mark of a student appeared in semester-1 and semester-2 examination. Pavani et al (2012) have evaluated the teachers' performance on the basis of fuzzy inference system (FIS), which is the process of formulating the mapping from a given input to an output using fuzzy logic. Bai and Chen (2008) have studied the difficulty and im-

portance and complexity of questions for students' answer script evaluation. Chen and Li (2011) have provided a useful way for students learning achievement evaluation based on the eigen-vector method. Chen and Wany (2009) have proposed a method which is more stable than Biswas method for students' answer script evaluation. Taylan and Karagozogly (2009) have proposed the class of Neuro fuzzy networks, with the ultimate aim to design a fuzzy inference system (FIS) Via learning. Chen and Lee (1999) have proposed a method which can overcome the drawbacks in the Biswas method due to fact that they do not need to perform the complicated matching operations and they can evaluate students answerscripts in a more fair manner.

## 2 The Study of Method

### Classical Methods

Identify a universe of discourse, X as a collection of objects all having the same characteristics. The individual elements in the universe X will be denoted as X. The features of the elements in X can be discrete, countable integers or continuous valued quantities on the real line. For Example, We have a universe comprised of three elements,  $X = \{a, b, c\}$ , so the cardinal number is  $n_X = 3$ .

The power set is

$$P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\} \quad (1)$$

The cardinality of the power set, denoted  $n_{P(X)}$ , is found as

$$n_{P(X)} = 2^{n_X} = 2^3 = 8$$

Note that if the cardinality of the universe is infinite, then the cardinality of the power set is also infinity, i.e.  $n_X = \infty \Rightarrow n_{P(X)} = \infty$ .

### Fuzzy Set

A set X in which each element y has a grade of membership  $\mu_X(y)$  in the range 0 to 1, i.e. set membership may be partial e.g. if cold is a fuzzy set, exact temperature values might be mapped to the fuzzy set

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portance and complexity of questions for students' answer script evaluation. Chen and Li (2011) have provided a useful

**Membership Function**

A membership function is a curve that defines how each point in the input space is mapped to a membership value between 0 and 1. Sometimes the input space refers to the universe of discourse.

**Definition:**

For any set X, a member function of X is any function from X to the real unit interval [0, 1]. Membership functions on X represent fuzzy subsets of X represents a fuzzy set A is usually denoted by  $\mu_A$ . For an element x of X, the value  $\mu_A$  is called the membership degree of x in the fuzzy set A. The membership degree  $\mu$  quantifies the grade of membership of the element x to the fuzzy set A. The value 0 means that x is not a member of the fuzzy set. The values between 0 and 1 characterize fuzzy members which belong to the fuzzy set par

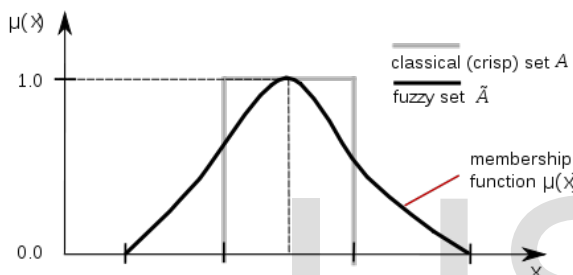


Figure 1: Membership function of a fuzzy set

**Trapezoidal fuzzy number:**

A trapezoidal fuzzy number A is a fuzzy number with a piecewise linear membership function  $\mu_A$  and is defined by

$$\mu_A(X) = \begin{cases} 0, & \text{When } X < a_1 \text{ and } X > a_4 \\ \frac{X - a_1}{a_2 - a_1}, & a_1 \leq X \leq a_2 \\ 1, & a_2 \leq X \leq a_3 \\ \frac{a_1 - X}{a_1 - a_3}, & a_3 \leq X \leq a_4 \end{cases} \quad (2)$$

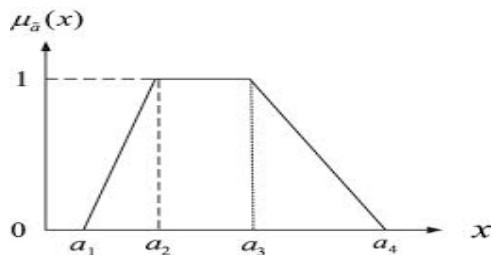


Figure 2: Trapezoidal Membership function of a fuzzy set

Trapezoidal fuzzy numbers are generally used when the fuzziness exists on both sides of an interval.

**Fuzzy Inference System**

Fuzzy inference systems (FISs) are also known as fuzzy rule-based systems, fuzzy model, fuzzy expert system and fuzzy associative memory. This is a major unit of a fuzzy logic system. The decision-making is an important part of the entire system. The FIS formulates suitable rules and based upon the rules the decision is made. This is mainly based on the concepts of the fuzzy set theory, fuzzy IF-THEN rules and fuzzy reasoning. FIS uses "IF... THEN..." statements, and the connectors present in the rule statement are "OR" or "AND" to make the necessary decision rules. The basic FIS can take either fuzzy inputs or crisp inputs, but the outputs it produce are almost always fuzzy sets.

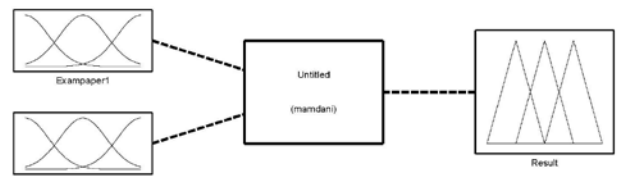


Figure 3: Determination of the student's performance

**3 FUZZIFICATION OF EXAM PAPER RESULTS AND PERFORMANCE VALUE**

Fuzzification of exam paper results was carried out using input variables and their membership functions of fuzzy logic system. Each input variable has five triangle membership functions.

Originally, membership functions have the equal interval, so both exam papers have an equal weighted comfortable. The fuzzy set of input variables is shown in Table 1.

Table 1. Fuzzy set of input variables

Linguistic Expression	Symbol	Interval
Very Low	VL	(0,0,20)
Low	L	(0,20,40)
Comfortable	C	(20,40,60)
High	H	(40,60,80)
Very High	VH	(60,80,100)
VeryVery High	VVH	(80,100,100)

It is seen that exam paper notes can belong to one or two membership functions but their membership weighting of each membership function can be different.

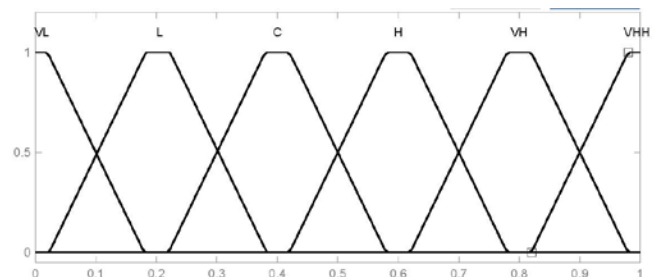


Figure 4. Membership functions of Exam paper 1 and Exam paper 2.

For instance, while a score of 20 only belongs to the “Low” membership function, a score of 30 belongs to both “Comfortable” membership functions, but is weighted more heavily within the “Low” membership functions than the “Comfortable” membership function.

The output variable, which is the performance value, is entitled “Result” and has six membership functions. For reasons of convenience within the application, a value range between 0 and 1 was chosen (Table 2 and Figure 3).

Table 2. Fuzzy set of output variables

Linguistic Expression	Symbol	Interval
Very Low	VU	(0,0,0.20)
Low	U	(0,0.20,0.40)
Comfortable	C	(0.20,0.40,0.60)
High	S	(0.40,0.60,0.80)
Very High	VS	(0.60,0.80,1)
Very Very High	VVS	(0.80,1,1)

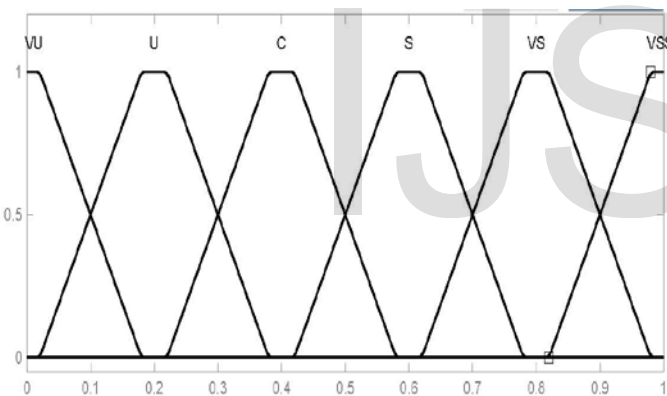


Figure 5 Membership functions of performance value

#### 4 FUZZY RULES AND INFERENCE

The rules determine input and output membership functions that will be used in inference process. These rules are linguistic and also are entitled “IF-THEN” Rules.

1. If Exam Paper 1 is VL and Exam Paper 2 is VL then Performance is VU
2. If Exam Paper 1 is VL and Exam Paper 2 is L then Performance is U
3. If Exam Paper 1 is VL and Exam Paper 2 is C then Performance is U
4. If Exam Paper 1 is VL and Exam Paper 2 is H then Performance is U
5. If Exam Paper 1 is VL and Exam Paper 2 is VH then Performance is C
6. If Exam Paper 1 is VL and Exam Paper 2 is VVH then Performance is S
7. If Exam Paper 1 is L and Exam Paper 2 is VL then Performance is VU
8. If Exam Paper 1 is L and Exam Paper 2 is L then Performance is U
9. If Exam Paper 1 is L and Exam Paper 2 is C then Performance is U
10. If Exam Paper 1 is L and Exam Paper 2 is H then Performance is C
11. If Exam Paper 1 is L and Exam Paper 2 is VH then Performance is C
12. If Exam Paper 1 is L and Exam Paper 2 is VVH then Performance is S
13. If Exam Paper 1 is C and Exam Paper 2 is VL then Performance is U
14. If Exam Paper 1 is C and Exam Paper 2 is L then Performance is U
15. If Exam Paper 1 is C and Exam Paper 2 is C then Performance is C
16. If Exam Paper 1 is C and Exam Paper 2 is H then Performance is S
17. If Exam Paper 1 is C and Exam Paper 2 is VH then Performance is S
18. If Exam Paper 1 is C and Exam Paper 2 is VVH then Performance is VS
19. If Exam Paper 1 is H and Exam Paper 2 is VL then Performance is U
20. If Exam Paper 1 is H and Exam Paper 2 is L then Performance is C
21. If Exam Paper 1 is H and Exam Paper 2 is C then Performance is S
22. If Exam Paper 1 is H and Exam Paper 2 is H then Performance is S
23. If Exam Paper 1 is H and Exam Paper 2 is VH then Performance is VS
24. If Exam Paper 1 is H and Exam Paper 2 is VVH then Performance is VSS
25. If Exam Paper 1 is VH and Exam Paper 2 is VL then Performance is C
26. If Exam Paper 1 is VH and Exam Paper 2 is L then Performance is S
27. If Exam Paper 1 is VH and Exam Paper 2 is C then Performance is S
28. If Exam Paper 1 is VH and Exam Paper 2 is H then Performance is VS
29. If Exam Paper 1 is VH and Exam Paper 2 is VH then Performance is VS
30. If Exam Paper 1 is VH and Exam Paper 2 is VVH then Performance is VSS
31. If Exam Paper 1 is VHH and Exam Paper 2 is VL then Performance is C
32. If Exam Paper 1 is VHH and Exam Paper 2 is L then Performance is S
33. If Exam Paper 1 is VHH and Exam Paper 2 is C then Performance is VS
34. If Exam Paper 1 is VHH and Exam Paper 2 is H then Performance is VS
35. If Exam Paper 1 is VHH and Exam Paper 2 is VH then Performance is VSS

36. If Exam Paper 1 is VHH and Exam Paper 2 is VVH then Performance is VSS

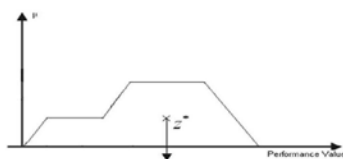
In case of several rules are active for the same output membership function, it is necessary that only one membership value is selected. This process is entitled fuzzy decision or fuzzy inference. Several authors, including Mamdani, Takagi-Surgeno and Zadeh have developed a range of techniques for fuzzy decision-making and fuzzy inference.

$$\mu_c(y) = \text{Max}(\text{Min}(\mu_A(\text{input}(i)), \mu_B(\text{input}(j))), k = 1, \dots, r \tag{3}$$

This expression concludes an output membership function value for each active rule. While one rule is active, an AND operation is applied between inputs. The smaller input value is selected and its membership value is determined as the membership value of the output of that rule. This method is repeated, so that performance membership functions are determined for each rule. To sum up, graphically AND (min) operations are applied between inputs and OR (max) operations are between performances.

### 5. DETERMINATION OF PERFORMANCE VALUE

There are several methods used for defuzzifying the fuzzy output functions. In this study, a Centroid (Center of Area) technique was applied, which is the most widely used methods. This can be called as center of gravity or center of area method. It can be defined by the algebraic expression is used for algebraic integration. The figure represents this method graphically.



$$Z = \frac{\int \mu_c(Z) x Z x dz}{\int \mu_c(Z) x dz}$$

### 6. APPLICATION OF FUZZY LOGIC IN PERFORMANCE

Table 3 Prove that the scores achieved by 20 students in Exam Paper1 and Exam Paper2. For each student, both exam scores were fuzzified by means of the membership functions previously described in Fuzzy Rules and Inference. Active membership functions were determined according to rule table, using the Mamdani fuzzy decision technique. The performance value was then defuzzified by calculating the center of the resulting geometrical shape. This sequence was repeat-

ed using the exam scores for each student.

Table 3. Calculate the Performance values and Exam Paper Scores

S. No	Exam Paper-1	Exam Paper-2	Performance	S. No	Exam Paper-1	Exam Paper -2	Performance
1	35	70	0.531	11	87	99	0.908
2	27	40	0.276	12	50	27	0.431
3	60	85	0.8	13	68	48	0.684
4	07	30	0.174	14	38	45	0.431
5	48	68	0.684	15	22	52	0.344
6	42	58	0.573	16	32	60	0.516
7	50	64	0.659	17	100	100	0.937
8	44	52	0.516	18	53	78	0.72
9	40	24	0.248	19	82	61	0.901
10	74	68	0.729	20	57	43	0.562

Both inputs had a same trapezoidal membership function. Therefore, replacing Exam Paper 1 and Exam Paper 2 would not change the calculated performance value (e.g. (48 & 68) and (68 & 48)). If the timekeeping or the value range of the membership functions is not equal, one of the exams has a greater influence on the output performance value than the other. For example, let's change the membership functions and value range or Exam Paper 2 (Figure 5) as retaining the original criteria for Exam paper1.

Table 4. Calculate the Performance values and Exam Paper Scores

S. No	Exam Paper-1	Exam Paper-2	Performance	S. No	Exam Paper-1	Exam Paper -2	Performance
1	35	70	0.542	11	87	99	0.591
2	27	40	0.259	12	50	27	0.865
3	60	85	0.627	13	68	48	0.784
4	07	30	0.200	14	38	45	0.500
5	48	68	0.784	15	22	52	0.490
6	42	58	0.448	16	32	60	0.5
7	50	64	0.519	17	100	100	0.33
8	44	52	0.745	18	53	78	0.758
9	40	24	0.719	19	82	61	0.64
10	74	68	0.47	20	57	43	0.292

The aim of this arrangement in Exam Paper 2 is to penalize scores below 50 and to reward scores above 50. This situation can be seen in Table 4.

For exam scores below 50, performance values decreased and for exam scores above 50, because this is the boundary of the limit value. Figure 6 shows the active rules and performance value obtained for exam scores of 48 and 68.



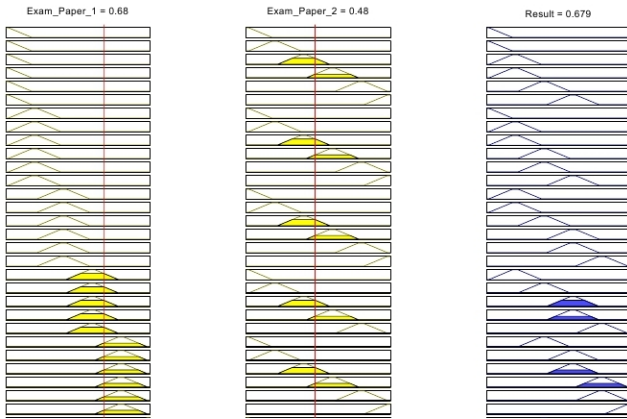


Figure 6. Active rules and performance value for exam scores of 48 and 68

In this scenario, rules 23, 24, 29 and 30 are active and at the end of defuzzification, a performance value of 0.679 is obtained.

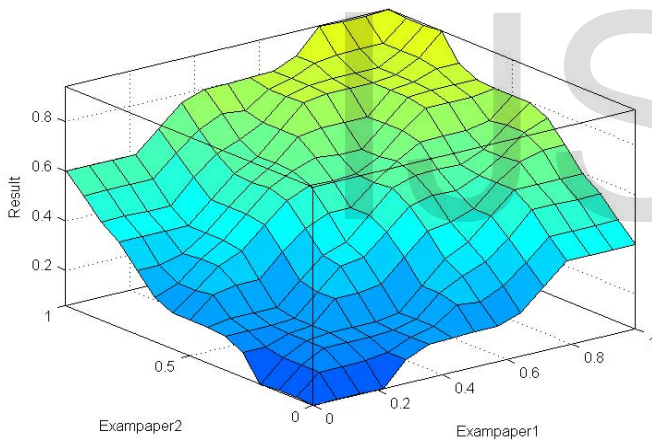


Figure 7: Surface viewer of academic performance evaluation.

## 7 COMPARISON OF CLASSICAL METHODS AND FUZZY LOGIC METHODS

Table 5. Calculate the Performance values and Exam Paper Scores

S. No	Exam Paper-1	Exam Paper-2	Classical Methods	Fuzzy logic 1	Fuzzy logic 2
1	35	70	0.525	0.531	0.542
2	27	40	0.335	0.276	0.259
3	60	85	0.725	0.8	0.627
4	07	30	0.185	0.174	0.200
5	48	68	0.58	0.684	0.591
6	42	58	0.5	0.573	0.448
7	50	64	0.570	0.659	0.519
8	44	52	0.48	0.516	0.745
9	40	24	0.32	0.248	0.719
10	74	68	0.71	0.729	0.47
11	87	99	0.93	0.908	0.591
12	50	27	0.385	0.431	0.865
13	68	48	0.58	0.684	0.884
14	38	45	0.415	0.431	0.500
15	22	52	0.37	0.344	0.490
16	32	60	0.46	0.516	0.5
17	100	100	1	0.937	0.33
18	53	78	0.655	0.72	0.758
19	82	61	0.715	0.901	0.64
20	57	43	0.50	0.562	0.292

Figure 8: Comparison of Classical Method, Fuzzy-1 and Fuzzy-2 for student academic performance

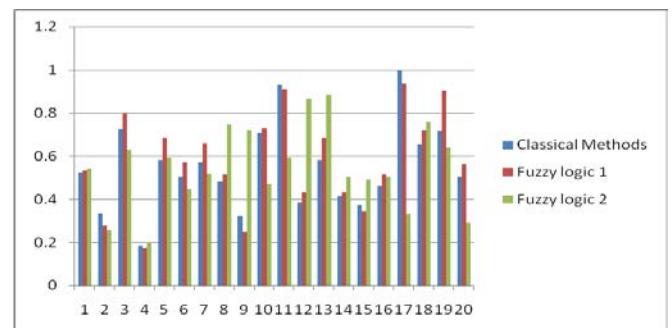


Figure 8: Comparison of Classical Method, Fuzzy-1 and Fuzzy-2 for student academic performance.

## 8 CONCLUSION

Performance values using the classical method and fuzzy logic method are given in the table 5. For comparison, average scores with classical method is divided into 100 and the success limit is accepted as 0.5. From table 5, a linear relationship can be seen between the classical method and fuzzy 1. If a student is successful in the classical assessment method, they will

also be successful in the fuzzy 1 scenario. Comparison of the classical method with the fuzzy 2 scenario reveals a difference in the performance values. For scores below 50, the performance value of fuzzy 2 is smaller than the classical method; however, for scores above 50, the performance value is larger than the classical method. For example, a student scoring 32 in Exam paper 1 and 60 in Exam paper 2 is unsuccessful in the classical method.

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